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A
NEW, CONCISE, AND CORRECT OPERATION
FOR CLEARING THE
Apparent Distance of the *MOON*
FROM
A *STAR* OR THE *SUN*
OF THE
EFFECTS OF REFRACTION AND PARALLAX:

Fully Exemplified, Illustrated and Demonstrated.

To which is added,

A

FREE AND IMPARTIAL EXAMINATION

OF

AN *ORIGINAL* PROJECTION FOR THE SAME PURPOSE,

As shewn in the

Practical Introduction to Spherics and Nautical Astronomy.

By H. CLARKE,

Author of the *SEAMAN's DESIDERATA*, &c. &c.

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BRITISH MUSEUM

A
NEW, CONCISE AND CORRECT OPERATION
FOR CLEARING THE
Apparent Distance of the MOON, &c.

A plain Illustration of the concise Operation in FORMULA I. *for clearing the Lunar Diſtances from the Effects of Parallax and Refraction, by Examples.*

EXAMPLE I. Fig. 2.

The rad. CD, from a decimal line of *equal parts*, is 58′ 45″=58·75, Moon's hor. par.
Zz is the arc of the app. diſt. of the centres - - 81° 23′ 38″
DL, Moon's alt. 48° 22′ 0″, P= ·777 } *Tab.*
EP, Sun's alt. 27 43 0, Q= ·491 } B. A.
Firſt Corr. MN, line *equal pts*. is 20·9 - - — 20 54
Second Corr. as ·777×58¾ : 60″ :: 20·9 : + 27
SN, line *equal pts*. is 40.
Third Corr. as ·491×58¾ : 60″ :: 40 : + 1 23
T=1·2, R=5·9, V=(39′) 19. *Tab*. D. C.
Fourth Corr. as 90 : 9·6 :: 13·1 : + 1
SN ÷ 7 = 40 ÷ 7 = — 5

True Diſtance 81 4 30

EXAMPLE II. Fig. 3.

The rad. CD, line *equal pts.* is 54′ 47″ = 54·78, Moon's hor. par.				
Zz is the arc of the app. diſt. - - -		41°	59′	57″
DL, Moon's alt. 42° 11′ 48″, P = ·71.				
EP, Sun's alt. 14 32 33, Q = ·268.				
Firſt Corr. MN, line *equal pts.* is 20·45,	+		20	27
Second Corr. as ·71 × 54·78 : 60″ :: 20·45 : SN, line *equal pts.* is 40.	−			31
Third Corr. as ·268 × 54·78 : 60″ :: 40 : T = 9·1, R = 5·5, V = (40′) 20.	+		2	43
Fourth Corr. as 90 : 72·8 :: 14·5 :	+			11
SN ÷ 7 = 40 ÷ 7 =	−			5
True Diſtance		42	22	42

EXAMPLE III. Fig. 4.

The rad. CD, line *equal pts.* is 59′ 39″ = 59·65, Moon's hor. par.				
Zz is the arc of the app. diſt. - - -		113°	14′	56″
DL, Moon's alt. 24° 48′ 0″, P = ·443.				
EP, Sun's alt. 14 11 57 Q = ·262.				
Firſt Corr. MN, line *equal pts.* is 27	−		27	0
Second Corr. as ·443 × 59·65 : 60″ :: 27 : SN = 35·2.	+		1	1
Third Corr. as 262 × 59·65 : 60″ :: 35·2 T = 3·6, R = 10·1, V = (54′) 37.	+		2	15
Fourth Corr. as 90 : 28·8 :: 26.9 :	−			8
SN ÷ 7 = 35·2 ÷ 7 =	−			5
True Diſtance		112	50	59

EXAMPLE

EXAMPLE IV.

From Mr. Mackay on the Long. p. 118. Fig. 5.

The rad. CD, is 59′ 17″ = 59·283, Moon's hor. par.				
Zz is the arc of the app. diſt. - - -		68°	35′	40″
DL, Moon's alt. 30° 44′ 0″, P = ·538.				
EP, Sun's alt. 33 31 0, Q = ·582.				
Firſt Corr. MN, is 23·3	−		23	18
Second Corr. as ·538 × 59·283 : 60″ :: 23·3 :	+			44
SN = 19·5.				
Third Corr. as ·582 × 59·283 : 60″ :: 19·5 :	+			34
T = 3·1, R = 7·4, V = (51′) 33.				
Fourth Corr. as 90 : 24·8 :: 25·6 :	+			7
SN ÷ 7 =	−			3
True Diſtance		68	13	44

EXAMPLE V.

From the Requiſite Tables, p. 41. Fig. 6.

The rad. CD, is 60′ 5″ = 60·083, Moon's hor. par.				
Zz is the arc of the app. diſt. - - -		50°	8′	41″
DL, Moon's alt. 55° 56′ 0″, P = ·88.				
EP, Star's alt. 19 18 0, Q = ·351.				
Firſt Corr. MN, is 15·75	+		15	45
Second Corr. as ·88 × 60·083 : 60″ :: 15·75 :	−			17
SN = 48.				
Third Corr. as ·351 × 60·083 : 60″ :: 48 :	+		2	16
T = 6·2, R = 3.6, V = (34′) 14·5.				
Fourth Corr. as 90 : 49·6 :: 10·9 :	+			6
True Diſtance		50	26	31

EXAMPLE VI.

From the Requisite Tables, p. 31. Fig. 7.

The rad. CD is 58′ 35″ = 58·583, Moon's hor. par.				
Zz is the arc of - - - - - -		103°	29′	27″
DL, Moon's alt. 41° 6′ 2″, P = ·697.				
EP, Sun's alt. 19 3 36, Q = ·347.				
Firſt Corr. MN, is 28·85	—		28	51
Second Corr. as ·697×58·583 : 60″ :: 28·85 : SN = 44.	+			43
Third Corr. as ·347×58·583 : 60″ :: 44 : T = 2, R = 10·5, V = (44′) 24·8.	+		2	9
Fourth Corr. as 90 : 16 :: 14·3 :	—			2
SN÷7 = 44÷7 =	—			6
True Diſtance		103	3	20

EXAMPLE VII.

From the Requisite Tables, p. 57.

The rad. CD is 54′ 42″ = 54·7. Moon's hor. par.				
The conſtr. being made with the app. diſt. - -		43°	35′	42″
And the Moon's alt. 9° 38′ 0″, P = ·18.				
Star's alt. 11 17 0, Q = ·22.				
Firſt Corr. MN, is 5·9	—		5	54
Second Corr. as ·18×54·7 : 60″ :: 5·9 : SN = 2·25	+			36
Third Corr. as ·22×54·7 : 60″ :: 2·25 : T = 8·5, R = ·5, V = (54′) 37.	+			11
Fourth Corr. as 90 : 68 :: 36·5 :	+			27
True Diſtance		43	31	2

A Demonſtration

A Demonstration of the Concise Operation for clearing the LUNAR DISTANCES *from the Effects of Parallax and Refraction, as given in* FORMULA I.

Fig. 1. is the well-known conſtruction for this purpoſe exhibited in the writings of M. *De la Caille* and M. *De la Lande*, and farther illuſtrated by the latter author in his Aſtronomy; afterwards adopted by Mr. *Mackay* in his accurate and uſeful treatiſe on the Longitude, and by ſeveral other writers,—in which HD repreſents the horizon, Z the zenith, DL the apparent altitude of the Moon, and LL its almicantar: alſo Zz denotes the apparent diſtance of the objects, EP the apparent altitude of the Sun or Star, and PP its almicantar. Moreover, CD expreſſes the Moon's given horizontal parallax; and MN, the principal effect of the Moon's parallax in diſtance, or the Firſt Correction, ſubtractive or additive as it falls to the right or left of CZ. Conſequently if CD be expreſſed in *minutes*, MN will be obtained in the ſame denomination.*

This part of the operation of finding the *First Correction* is therefore preciſely the ſame with that ſhewn by the various Authors juſt mentioned. The inveſtigation of this will not be obſcure, if we only conceive a ſpheric triangle to be formed of the apparent co-altitudes of the objects as ſides (having its vertical or included angle in the zenith) and the obſerved diſtance as the baſe, that is. of the arcs ZL the co-altitude of the Moon, zP the co-altitude of the Sun or Star, and Zz the apparent diſtance; the parallax in diſtance will then be expreſſed by the product of the Moon's parallax in altitude (DL) into the coſine of the angle formed by the ſpheric triangle at the Moon L; which is demonſtrated by every ſcientific writer on Aſtronomy. The ſame alſo holds good in reſpect to refraction. Now, the parallax of the Moon in altitude is as the coſine of its apparent altitude; therefore if CD repreſent the Moon's horizontal parallax, ML, the coſine of DL the Moon's altitude, will expreſs, or be analogous to, the parallax in altitude. And ſince MN is the coſine of the angle oppoſite to zP in the imaginary ſpheric triangle above alluded to, that is, the coſine of the angle at the Moon (*vid. De la Lande*'s Aſtron. tom. 3. p. 723), it is then equal to the parallax in altitude multiplied by the coſine of the angle at the Moon, eſteeming ML as rad. 1. that is, to the parallax in diſtance. If therefore the radius CD in the conſtruction be taken equal to the Moon's given horizontal parallax in minutes from a ſcale of equal parts, MN meaſured on the ſame ſcale, will give the *First Correction* alſo in minutes, which is the principal effect of the Moon's parallax in diſtance, and is ſubtractive or additive as the angle at the Moon in the ſpheric triangle alluded to is acute or obtuſe. In the former caſe MN falls to the right, and in the latter to the left of CZ. Conſequently if the angle at the Moon be a right angle, this correction vaniſhes. Hence the firſt part of M. *De la Caille's Chasis de Reduction*; or Mr. *Mackay*'s ſcale I. plate V. p. 130.

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* It is ſcarcely neceſſary to obſerve that the conſtruction for MN is exactly the ſame, if we draw only CZ, Cz in the given angle, taking CM, CS, the ſines of the arcs DL, EP, and drawing the perpendiculars ML, SP. But this would evidently be leſs accurate, on account of the ſines; and more trouble, by drawing the perpendiculars.

By the ſame reaſoning, MN will be expreſſive of the effect of the Moon's refraction in diſtance. But as the refractions are not governed by the ſame laws as parallax, it is neceſſary, in order that ML may have a proper relation to, or truly repreſent, the refraction in altitude, that CD expreſs the refraction of the altitude divided by the coſine of the altitude; then will MN, the part of ML, expreſs the refraction of height (ML being accounted rad. 1) multiplied by the coſine of the angle at the Moon, and conſequently give the true value of the Moon's refraction in diſtance. Therefore, ſince the refractions are found at different altitudes to be in the inverſe ratio of the coſine of the altitude, we ſhall have, by aſſuming any determinate horizontal parallax, ſuppoſe 60′, the following theorem for conſtructing a ſcale or table of the Corrections for refraction in diſtance, which is again ſimilar to M. *De la Caille's Chasis de Reduction*, or Mr. *Mackay*'s ſcale 2d. plate V.

$$\frac{60 \times \textit{nat. coſine alt. rad. } 1}{\textit{Refract. of alt in seconds}} = P,$$

and from whence, by I. *Req. Tables*, the numbers in the annexed Tables in the *Formula* have been computed. This table is therefore adapted to the Moon's hor. par. 60′, and to 1 minute or 60 ſeconds of radius. Hence to reduce the value of MN to any given hor. par. it will be

as CD (given par.) : 1 :: MN (firſt corr.) : $\frac{MN}{CD}$,

and as P : 60″ :: $\frac{MN}{CD}$: $\frac{60MN}{P \times CD}$, that is,

as P×CD : 60″ :: MN : the Second Corr. in *seconds*;

P being the number in the Table correſponding to the Moon's altitude. If the angle at the Moon be obtuſe, the corr. is ſubtractive; otherwiſe additive: If a right angle this correction vaniſhes.

The *Third Correction* for the Sun's or Star's refraction is determined preciſely in the ſame manner; SN having the ſame relation to EC, as MN has to CD. That is,

$\frac{60\times SN}{Q\times CD} =$ (or Q×CD : 60″ :: SN :) the third corr. in *ſeconds*;

Q being the tab. numb. anſwering to the Sun's or Star's altitude. If the angle at the Sun or Star be acute, SN falls to the left of Cz, and is additive; otherwiſe ſubtractive: If a right angle this corr. vaniſhes.

The Table for the *Fourth Correction* in the *Formula* is computed by means of this theorem,

$$\frac{4pv \times \text{Sq. root. } \overline{1-pp}}{1-2pp},$$

in which *v* is the Moon's par. in altitude, or in diſtancce, and *p* the ſine of one fourth the apparent altitude or diſtance to radius 1. Hence may Mr. *Mackay*'s ſcales Nos. 4 and 5 be conſtructed. This correction may however be computed by different methods, which will be obvious by conſidering the properties of the ſpheric triangles from whence it originates; and from whence alſo it will appear that if the diſt. exceed 90 deg. the corr. will be ſubtractive; otherwiſe additive.

The correction for the Sun's parallax depends, like corr. third, on the value of SN, and will be found from computation to be nearly in the ratio aſſigned; the error ſcarcely ever amounting to one ſecond; and is additive or ſubtractive as the angle at the Sun is obtuſe or acute.

OCCASIONAL.

OCCASIONAL REMARKS.

ON my firſt publication of *The Seaman's Deſiderata*, the principal deſign of which was to bring forward to the MARINER—*A New and Complete* FORMULA *for expediting the Computation of the Longitude at Sea by means of the Lunar Diſtances*, including a *Conciſe and Correct Mode of Calculation* for clearing thoſe diſtances from the effects of Parallax and Refraction without the uſe of *verſed ſines*, as given in p. 27, of that treatiſe,—I caſually introduced, among other matter, a kind of *approximate* method for the latter purpoſe, which was intended to be uſed occaſionally (within certain limits) merely as a rough expeditious proof or check to the more complex proceſſes by computation; or, as a ſubſtitute for them when no extreme nicety was requiſite. And to this I conceived I had an undoubted right, as the conſtructive part was manifeſtly the ſame with that ſhewn by M. *De la Caille*, M. *De la Lande*, Mr. *Mackay*, and ſeveral other writers, and the ſubſequent or computative part immediately deducible from an approximate rule that had been diſcovered upwards of twenty years ago, and ſince that time publicly taught to ſome hundreds of perſons in Mancheſter, Liverpool, Briſtol, &c. It is now near twenty-five years ſince the late Mr. *Jeremiah Ainſworth* (a Gentleman well known to the mathematic world by his very extraordinary abilities) and myſelf, attempted to inveſtigate *a conciſe practical method* of performing the problem under conſideration at ſea. On examining the conſtruction above adverted to, it required no great ſagacity to perceive, that as CD was the Moon's hor. par. in minutes, which was always between the limits of 62 and 53, that a line of chords of the ſame radius with CD would not differ much in its diviſions, at leaſt in the former part of the line, from the line of equal parts, calling every degree a minute. And accordingly it appeared that when the Moon's parallax was at about a mean (57½ min.), the *chords* might be ſubſtituted for the *equal parts* to one-half the line or more, without occaſioning any material error. But when the value of MN became conſiderable, and the Moon's par. exceeded the mean, the chords gave the value of MN too little. On the other hand, if the Moon's par. was leſs than the mean, MN was found to be too great by the chords. It remained now, therefore, to find a conciſe approximation that would not only counteract theſe errors ariſing from the line of chords, but, at the ſame time, would compenſate for the remaining corrections owing conjointly to the refractions of both the obſerved bodies, and the parallax of the Sun (if uſed). The only ſimple method of effecting this appeared to be by a mode of induction. Several ſeries of Examples were accordingly accurately calculated, principally by the rules given in the Nautical Almanacs of 1767 and 1772, with the Moon's hor. par. at the intervals of half a minute each, taken both ways from the mean to its utmoſt limits, and the *whole correction* of each example noted down in regular order. The *Firſt Correction* in each example was then found by this conſtruction and the line of chords; and, from a careful and extremely ſcrupulous compariſon of the whole of the reſults in a variety of aſſumptions with this correction (the quantity of which evidently depended on the obſerved altitudes and diſtance of the objects) and the conſtant or known quantities as *data*, which could be no other than the Moon's greateſt, leaſt, and given hor. parallaxes, it appeared at length that a near value of the *whole correction of distance* would in general be obtained from the expreſſions

$$-\frac{ab}{p},\ \text{and}\ +\frac{ab}{q},$$

in which a is the *first* corr. from the line of chords, b the Moon's given hor. par. and p the greateſt (62′), and q the leaſt parallax (53′). The former being applied when a is ſubtractive, and the latter when a is additive, which gives this remarkable rule,

When the first corr. is —, *multiply it by the Moon's given hor. par. and divide the product by* 62, *the quotient is the* whole correction *subtractive. But if the first corr. be* +, *divide the same product by* 53, *the quot. is the* whole correction *additive.**

Ex. 1. Given $a=36$, $b=58$, $p=62$; hence $-ab\div p=-33'.\ 40''$, the whole correction. See Naut. Aſtrom. p. 203.

Ex. 2. Given $a=22$, $b=54{\cdot}75$, $q=53$; hence $+ab\div q=+22'.\ 43''$, the whole correction. See p. 44 of this treatiſe.

On reſuming this ſubject ſome few years after, with a view to adapt a proper correction to this approximation when MN had a conſiderable value, the chords then differing ſo much from the equal parts or proper meaſures of CD, as to occaſion an error in the expreſſions for the *whole correction*, which rendered the rule uſeleſs in ſuch caſes, and by no means to be depended on at ſea; it was found, from a method too tedious to be pointed out here, that the expreſſion

$\frac{b}{p}.a$ was in general too little by the quantity $\frac{p-b}{pp+p}$, and $\frac{b}{q}.a$ too great by the quantity $\frac{b-q}{qq+q}$; the chords giving the value of MN too little when —, and too great when +. But $\frac{b}{p}+\frac{p-b}{pp+p}$ is $=\frac{b+1}{p+1}$, and $\frac{b}{q}-$

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* This rule with its proper limitations, was inſerted by Mr. *Ainsworth* and myſelf in the Liverpool papers about the year 1776.

$\frac{b-q}{qq+q}$ is $=\frac{b+1}{q+1}$; hence the former rule ſtood corrected $-\frac{b+1}{p+1}\cdot a$, and $+\frac{b+1}{q+1}\cdot a$. That is,—*To the Moon's given hor. parallax, and also to its greateſt parallax if* a *be* —, *or leaſt if* a *be* +, *add* 1. *Divide the former ſum by the latter; and multiply the quotient by the firſt correction: the product is the* whole correction *of the apparent diſtance, which is additive or ſubtractive, according as the value of* a *is* + *or* —.

This corrected rule I have taught alſo to my pupils for theſe twelve years and upwards as an uſeful approximation; but not in all caſes to be abſolutely depended on, having its defects in ſome meaſure as the former, which I ſhall preſently point out. Theſe rules, however, were found to be ſo convenient at ſea, when no great accuracy was required, or by way of check to the more operoſe computations, that about four or five years ago I was ſo much preſſed by ſeveral ſeafaring Gentlemen to publiſh them, I drew up a few *Formulæ* adapted to facilitate thoſe calculations that are in general uſe among ſeamen, and along with theſe this conciſe method of clearing the Lunar Diſtances, accompanied with examples, nearly in the form they now make their appearance. But before I had properly prepared them for the preſs, a new publication on the ſubject of Nautical Aſtronomy, exhibited a method of performing the problem of the Lunar Diſtances in many reſpects ſimilar to my own: the conſtructive part being eſſentially the ſame with that I had adopted from the authors before-mentioned, and the computative part appearing to be only a different mode of expreſſing the very ſame operation as mine. In this latter reſpect I was ſoon convinced, for in reducing the expreſſion $-\frac{b+1}{p+1}\cdot a$ to $+\frac{\overline{p-b}\cdot a}{p+1}-a$, it was evident, that as this denoted the *whole* correction, and $-a$ was in this caſe the *firſt* correction from the chords, the computative or *ſecond* part of the corr. muſt be given by $+\frac{p-b\cdot a}{p+1}$; hence obviouſly aroſe this analogy, as $p+1 : p-b :: a :$ 2d. corr. that is, as $63 : 62-b :: a :$ ſecond correction: The rule I lately gave for the firſt caſe in the *Formula*. In like manner $+\frac{b+1}{q+1}\cdot a$ appeared to be reducible to $+\frac{\overline{b-q}\,.\,a}{q+1}+a$; hence, in this caſe alſo, $+a$ is the *first* correction, and the *second* is had from this analogy, as $q+1 : b-q :: a :$ 2d. corr. that is, as $54 : b-53 :: a :$ ſecond correction; the rule which was given in the ſecond caſe in the *Formula*.

Now it is very evident that other forms, by contraction, &c. of the above expreſſions, or of their deduced analogies, may be eaſily obtained; as for inſtance, by dividing the antecedents, or conſequents, by ſome number of which one or both of them may be a multiple, &c. But of what uſe or advantage is this, unleſs it be for the ſake of apparent originality?—The analogies are certainly more ſimple in their firſt form, as it will ſeldom happen that the other term is diviſible by the ſame number without a remainder. The deduced analogies are

$63 : 62-b :: a :$ 2d. corr. and $54 : b-53 :: a :$ 2d. corr.

Here 63 is diviſible by 7 or 9; therefore

$9 : 62-b :: \frac{a}{7}$ 2d. corr. or, $7 : 62-b :: \frac{a}{9}$: the ſame corr.

And 54 is diviſible by 6 or 9; hence

$9 : b-53 :: \frac{a}{6} :$ 2d. corr. or, $6 : b-53 :: \frac{a}{9}$: the ſame corr.

And would not the dividing by any other number in like manner bring out the ſame concluſion?—This every ſchool-boy is convinced of, if he has but attained the rule of proportion; and therefore the forming of the analogy by ſaying—"as 9, *the difference of the greatest and least horizontal parallax, &c.*"—is merely *vox et prœterea nihil*—an unmeaning expreſſion, evidently introduced in this place to give the proceſs an air of ſcientific myſtery; as it would have been as much to the purpoſe, and with equal propriety, to have ſaid, as 9, *the number of the musical nymphs*, &c. or, as 7, *the number of wise men of the East,—the renowned champions of christendom*, &c. &c.——But enough,—how oft is the ſentiment of the Poet verified—*Decipimur specie recti!*

$\frac{b-q}{qq+q}$ is $= \frac{b+1}{q+1}$; hence the former rule ſtood corrected $- \frac{b+1}{p+1} . a$, and $+ \frac{b+1}{q+1} . a$. That is, — *To the Moon's given hor. parallax, and also to its greatest parallax if* a *be* —, *or least if* a *be* +, *add* 1. *Divide the former sum by the latter; and multiply the quotient by the first correction: the product is the* whole correction *of the apparent distance, which is additive or subtractive, according as the value of* a *is* + *or* —.

This corrected rule I have taught alſo to my pupils for theſe twelve years and upwards as an uſeful approximation; but not in all caſes to be abſolutely depended on, having its defects in ſome meaſure as the former, which I ſhall preſently point out. Theſe rules, however, were found to be ſo convenient at ſea, when no great accuracy was required, or by way of check to the more operoſe computations, that about four or five years ago I was ſo much preſſed by ſeveral ſeafaring Gentlemen to publiſh them, I drew up a few *Formulæ* adapted to facilitate thoſe calculations that are in general uſe among ſeamen, and along with theſe this conciſe method of clearing the Lunar Diſtances, accompanied with examples, nearly in the form they now make their appearance. But before I had properly prepared them for the preſs, a new publication on the ſubject of Nautical Aſtronomy, exhibited a method of performing the problem of the Lunar Diſtances in many reſpects ſimilar to my own: the conſtructive part being eſſentially the ſame with that I had adopted from the authors before-mentioned, and the computative part appearing to be only a different mode of expreſſing the very ſame operation as mine. In this latter reſpect I was ſoon convinced, for in reducing the expreſſion

$- \frac{b+1}{p+1} . a$ to $+ \frac{\overline{p-b} . a}{p+1} - a$, it was evident, that as this denoted the *whole* correction, and $-a$ was in this caſe the *first* correction from the chords, the computative or *second* part of the corr. muſt be given by $+ \frac{\overline{p-b} . a}{p+1}$; hence obviouſly aroſe this analogy, as $p+1 : p-b :: a :$ 2d. corr. that is, as $63 : 62-b :: a :$ ſecond correction: The rule I lately gave for the firſt caſe in the *Formula*. In like manner $+ \frac{b+1}{q+1} . a$ appeared to be reducible to $+\frac{\overline{b-q}\,.\,a}{q+1}+a$; hence, in this caſe alſo, $+a$ is the *firſt* correction, and the *ſecond* is had from this analogy, as $q+1 : b-q$ :: a : 2d. corr. that is, as $54 : b-53 :: a$: ſecond correction; the rule which was given in the ſecond caſe in the ***Formula***.

Now it is very evident that other forms, by contraction, &c. of the above expreſſions, or of their deduced analogies, may be eaſily obtained; as for inſtance, by dividing the antecedents, or conſequents, by ſome number of which one or both of them may be a multiple, &c. But of what uſe or advantage is this, unleſs it be for the ſake of apparent originality?—The analogies are certainly more ſimple in their firſt form, as it will ſeldom happen that the other term is diviſible by the ſame number without a remainder. The deduced analogies are

$$63 : 62-b :: a : \text{2d. corr. and } 54 : b-53 :: a : \text{2d. corr.}$$

Here 63 is diviſible by 7 or 9; therefore

$$9 : 62-b :: \tfrac{a}{7}\ \text{2d. corr. or, } 7 : 62-b :: \tfrac{a}{9} : \text{the ſame corr.}$$

And 54 is diviſible by 6 or 9; hence

$$9 : b-53 :: \tfrac{a}{6} : \text{2d. corr. or, } 6 : b-53 :: \tfrac{a}{9} : \text{the ſame corr.}$$

And would not the dividing by any other number in like manner bring out the ſame concluſion?—This every ſchool-boy is convinced of, if he has but attained the rule of proportion; and therefore the forming of the analogy by ſaying—"as 9, *the difference of the greateſt and leaſt horizontal parallax, &c.*"—is merely *vox et præterea nihil*—an unmeaning expreſſion, evidently introduced in this place to give the proceſs an air of ſcientific myſtery; as it would have been as much to the purpoſe, and with equal propriety, to have ſaid, as 9, *the number of the muſical nymphs*, &c. or, as 7, *the number of wiſe men of the Eaſt,—the renowned champions of chriſtendom*, &c. &c.——But enough,—how oft is the ſentiment of the Poet verified—*Decipimur ſpecie recti!*

Cautions to SEAMEN *respecting the Concise Method of clearing the* LUNAR DISTANCES, *as given in the Treatise adverted to; and which were intended to have been added as a necessary Note to p.* 43, *of the* SEAMAN'S DESIDERATA, *had the first Impression of it not been suppressed.*

LEST any mariner ſhould implicitly rely on, or put too great confidence in this method of clearing the Lunar Diſtances, it has been thought proper to point out to him its ſeveral imperfections, and the inaccuracies to which it is liable.

By this method it appears that the eſtimated or *second* correction has ſuch a dependance on the *first*, that, whatever the difference of the greateſt or the leaſt and the given hor. parallax of the Moon may be, whenever the latter correction vaniſhes or becomes *o*, as it evidently muſt do when the angle at the Moon in the ſpheric triangle before-mentioned is 90 degrees (the coſine being then = *o*, and the points M and N in the conſtruction coincide) the ſecond or eſtimated part of the correction muſt vaniſh, or become *o* alſo. Here then is one glaring proof of the incorrectneſs of this method, and of its inconſiſtency with true principles. For the value of SN, which is expreſſive of the refraction of the object from which the Moon's diſtance is obſerved, exiſts entirely independent of the quantity of MN, and has, in this caſe, nothing to counteract, or diminiſh it, but the inconſiderable effect of the Sun's parallax in diſtance, if that object be taken; except indeed the ſmall *fourth* correction, when it happens to be affected with a contrary ſign to that of SN; which, however, can never be equal to it, unleſs under the occurrence of ſuch particular circumſtances as ſcarcely ever happen in practice. The following inſtance will fully juſtify this obſervation.

GIVEN	The Moon's diſt. from the Sun	68°	35′	40″	Req. the true diſtance
	Sun's alt.	19	19	0	
	Moon's alt.	65	0	0	
	Moon's hor. par.	0	59	17	

In making the conſtruction, the points M and N will be ſound exactly to coincide;* hence, by this method, there is no correction neceſſary, and the apparent diſtance of the centres is the true diſtance, *viz.* 68° 35′ 40.″

Let us now from theſe *data* find the true diſtance by the new operation given in *Formula* I. From the ſame conſtruction MN is = *o*; but SN is found to be 50·4, from the ſame line of equal parts that CD is 59·283, the Moon's hor. parallax.

		°	′	″
Therefore, App. diſt.		68°	35′	40″
Firſt corr. -		—	—	—
Second corr.		—	—	—
SN = 50·4, Q = ·352 (Sun's alt. 19° 19′)				
Third corr. as ·352×59 283 : 60 :: 50·4 :	+		2	25
T = 3·1. R = o, V = (25′) 8·4,				
Fourth corr. as 90 ; 24·8 :: 8·4 :	+			2
SN ÷ 7 =	—			7
True diſtance,		68	38	0

As a further confirmation, let the true diſtance be *computed* from the *Formula* p. 27.

	°	′	″			°	′	″
Corr. Moon's alt.								
App. alt. centre	65	0	0					
Hor. par.		59	17		+	0	24	37
Corr. Sun's alt. -				Par.	—			8
				Refr.	+		2	41
Sun's alt. leaſt					+	0	27	10
Moon's app. alt.	65°	0′	0″					
Sun's app. alt.	19	19	0					
Diff. app. alts.					+	45	41	0
Diff. true alts.						46	8	10

Diff.

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* It is evident from Fig. 1. that this circumſtance will always take place, when the coſine of the apparent diſtance is to the ſine of the Sun's (Star's) altitude, as rad. is to the ſine of the Moon's altitude; For in the triangle CMS, coſ. ang. C : CS :: rad. : CM.

Diff. true alts.	46°	8′	10″	N. C.	692947	
Diff. app. alts.	45	41	0	N. C.	698623	
App. diſt.	68	35	40	N. C.	364967	
				Diff.	333656	5·523299
Moon's alt. and hor. par. IX. R. T.						9·993391
				N. N.	328618	5·516690
True diſt.	68°	38′	1″	N. C.	364329	

Here then is an error of 2′ 20″ in the Lunar diſtance by the method under conſideration, which, according to the horary motion of the obſerved bodies, would occaſion an error in the Longitude from 60 to 70 miles.* And numbers of examples might in this caſe be given where the errors are much more conſiderable, amounting to even 150 miles and upwards; and this evidently does not ariſe from any inaccuracy or uncertainty in the conſtruction, but from the very principle itſelf; the corrections relating only to the Moon's parallax and refraction, which a bare inſpection of Fig. 5 will evince. For MN will have the ſame value, and the ſubſequent eſtimated correction will be preciſely the ſame (with any other diſtance as Z*x* and altitude *ep*) whether SN be *o*, or any quantity whatever.

Again, it follows from this method, that there can be no *ſecond* or ſubſequent correction to the *first*, when the Moon's given hor. parallax is at the *greateſt* or *leaſt*, as the difference between the given and the greateſt or the leaſt parallax then vaniſhes, or becomes = *o*, and conſequently the fourth term for the *eſtimated* correction becomes *o* alſo. Another glaring proof of the fallaciouſneſs of this method. For, as to arguing that the exceſs or defect of the value of MN by the chords, with reſpect to its true value by the line of equal parts (of which CD is the Moon's given hor. parallax in minutes) may correct the errors ariſing from hence, is vague and unſcientific, and wholly unſupported by any mode of reaſoning by approximation; as will clearly appear on a proper examination, and be fully confirmed by trial.* The following inſtance may be adduced to verify the aſſertion.

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* It may not be improper here to obſerve to the Mariner, that, at a mean, one minute of error in the apparent diſtance will occaſion an error in the Longitude inferred from thence of 27 miles. For the error reſulting in Longitude from an error in diſtance, is as the diurnal motion of the Moon to 360 degrees. Thus, ſuppoſe the Moon's diurnal motion to be 13° 10½′, 4 min. of error in diſtance will produce 1° 49′ error in Longitude.—*De la Lande.*

GIVEN	App. diſt.	81°	23′	38″	Req. the true diſtance.
	Moon's alt.	50	0	0	
	Star's alt.	10	0	0	
	Moon's hor. par.	62	0	0	

The conſtruction being made, and MN meaſured from the line of chords, the firſt correction will be found to be 3½, which is —. Therefore 81° 23′ 38″ — 3′ 30″=81° 20′ 8″, the *true diſtance* according to this method; for the ſecond corr. vaniſhes, being as 63 : 62—62 :: 3′ 30″ : 0.

Now, let the example be performed by the operation in *Formula* I. and we ſhall have MN = 3·7, and SN = 46·8, from the line of equal parts; and from the given altitudes, we find P = ·8035, Q = ·1876; alſo T = 1·16, R = 1·07, and V = (40′) 20. Hence App.

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* Any perſon who will take the trouble, or may think it worth his while, may ſoon be convinced of this by the following proceſs.—Let a line of chords be laid down (to 60 deg. will be ſufficient), and to the ſame radius or diſtance ſeveral lines of *equal parts* be adapted, which may meaſure the Moon's hor. parallax from 62 to 53, the greateſt and leaſt limits. Now, let ſeveral aſſumed values of MN be taken from the line of chords as the firſt correction, and the eſtimated corrections made according as MN is — or +, with the Moon's given hor. parallax, and the reſults of each correction placed in regular order. Let the ſeveral values of MN be now found from the lines of equal parts according to the Moon's hor parallax, and they will each expreſs the true corr. for the Moon's par. in diſtance; and from theſe ſeveral values of MN, let the refractions in diſtance be alſo found by the rule in *p.* 8, and each of theſe corrections be placed as before in ſuch manner ſo as to form a proper compariſon between the reſults of each. The various inaccuracies of this method will now appear in full view,—exceeding, or falling ſhort of the truth in the moſt irregular manner; and very ſeldom approaching to it, or coinciding therewith, but, as it were, by mere accident. And in many caſes, it is remarkable, that the ſecond correction makes it ſtill more erroneous. For when the Moon's parallax exceeds the mean (57½), the chords, in general, give the *firſt* corr, too little, and when leſs than the mean, too much; hence, if MN (*a*) be —, in the firſt caſe, the *whole* correction, which is expreſſed by $-\frac{b+1}{p+1}\,.\,a$ (*p.* 12) will evidently be ſtill leſs, ſince *p* the greateſt par. muſt exceed *b* the given parallax. And, in the latter caſe, if MN be +, the *whole* corr. $+\frac{b+1}{q+1}\,.\,a$ muſt be ſtill more, ſince *q* the leaſt parallax is leſs than *b*. This is on the ſuppoſition of there being no other corrections but what relate to the Moon: for it is evident, from what has been before remarked, that this method has no reſpect whatſoever to the corrections ariſing from the other object; and therefore, on this account alſo, if the reſult be tolerably near the truth, it muſt be by mere chance.——It would be endleſs to enumerate all the imperfections which a compariſon of this method with a true proceſs exhibits; and therefore, on the whole, it can only be accounted a caſual operation, to which no perfect rules can be adapted (for if ſuch were attempted they would be found to be infinitely more complex, intricate and embarraſſing than any thing that has hitherto appeared on the ſubject), depending on the value of MN, which being the principal correction and in *minutes*, and the other corrections in general but ſmall ones and in *seconds*, the ſubſequent eſtimation may ſometimes bring out a concluſion that is not much erroneous.

App. diſt.		81°	23′	38″
Firſt corr.	−		3	42
Second corr.	+			4
Third corr.	+		4	1
Fourth corr.	+			2
True diſtance		81	24	3

By computation alſo from *Formula* p. 27.

Corr. Moon's alt.	+	0°	39′	3″
Corr. Star's alt.	+	0	5	15
Star's alt. leaſt	+	0	44	18
Diff. app. alts.	+	40	0	0
Diff. true alts.		40	44	18

Diff. true alts.	40°	44′	18″	N. C.	757699	
Diff. app. alts.	40	0	0	N. C.	766044	
App. diſt.	81	23	38	N. C.	149641	
				Diff.	616403	5·789865
IX. R. T.						9·994171
				N. N.	608185	5·784036
True diſt.	81°	24′	4″	N. C,	149514	

Here again we find this method to be extremely incorrect, an error ariſing in the diſtance of nearly 4 minutes, occaſioning, at a mean, a wrong determination of the Longitude of upwards of 400 miles. And the reſults will be frequently found to be equally, if not more erroneons, when the Moon's hor. parallax is at or near the leaſt, 53 minutes. But this method becomes alarmingly more uncertain in every caſe when the altitudes of the obſerved objects are conſiderable, and are laid down by means of a *line of ſines* on the radius [*vid.* p. 7], 60, 70, and even 80 degrees or more of altitude being often unavoidable caſes at ſea. The uncertainty of this method by ſines is ſo great, that I have ſeen frequent inſtances of two or more perſons, who have been well ſkilled and equally accurate in practical mathematics, attempting the ſame example in this way, who have differed in their determination of the Longitude upwards of 250 miles, even when they have had the moſt correct ſcales for the buſineſs.

After all, it muſt be allowed, that this method, imperfect as it is, may, notwithſtanding, be a piece of uſeful knowledge to a Seaman in comparing his *computation* of diſtance, when it can be applied within its proper limits, of which he muſt now be fully apprized. Due regard being had to this, and the altitudes laid down in the uſual manner by the chords on the periphery, he will find the rules in p. 11, 12, generally give a reſult not much wide of the truth. But the New Operation for this purpoſe, I have now ſhewn in *Formula* I. is correct in all caſes, provided the conſtructive part be performed with care; and, with a little practice, will be found to be nearly as expeditious as the former. The *Formula* for computation in p. 27, *Seaman's Desiderata*, is, however, abundantly ſuperior to any conſtructive or graphic methods whatſoever, and ſhould always be preferred when perfect accuracy is required: and it is moreover ſo conciſe, that any Seaman, who is tolerably expert in figures, may go through the whole in leſs than twenty minutes.*

And now, I ſhall conclude theſe remarks, in congratulating the Mariner into whoſe hands that little treatiſe may fall, and myſelf alſo, on the *expunging* of *Formula* I. with pages 43, &c. as given in the firſt impreſſion of the *Desiderata*, as it has afforded an opportunity of examining and bringing to view, more fully than was at firſt intended, the merits and demerits of this method of clearing the Lunar diſtances, and of ſubſtituting others containing matter of much greater value and importance. It cannot, however, but be obſerved, that the circumſtances attending the occaſion of this extraordinary event, have not been of the moſt pleaſant nature; and, it is preſumed, it may with truth be ſaid, that it cannot add much credit to either the head or heart of the man who could adopt and purſue ſuch violent and oppreſſive meaſures in contending for the *honour* of a Diſcovery, which, in reſpect to *real* Science, is indeed little more than——*a bubble!*

O! miseras hominum mentes, oh! pectora cæca!

H. C.

BRISTOL, 1800, *Oct.* 1*st.*

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* The method now given as an Appendix to the Requiſite Tables by the *Astronomer Royal*, by means of *Tables*, is an excellent approximation; and though, like every other operation for this purpoſe from whence accuracy is to be expected, it may, on a bare inſpection, appear tedious; yet, it will be found by a little uſe, that it may be performed with great facility and expedition.

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